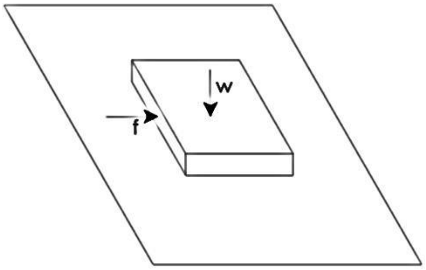
Belt Theory

Friction Drives - V-Belts and V-Ribbed Belts



The figure to the left illustrates the basic idea of static friction. If the block weighs "w" (perpendicular to the surface) and the force (parallel to the surface) "f" can be applied to the side of the block before it begins moving, then the coefficient of *static friction* $\mu_S = \frac{f}{w}$ after the block begins to slip, the force "f" can be reduced while still overcoming the *kinetic friction* $\mu_K = \frac{f}{w}$ V-belts and flat belts employ the use of friction to transmit power while timing belts physically engage the pulley via teeth and grooves. A friction drive consists of two shafts connected by a belt that is drawn tight enough to grip each shaft and transmit torque. Generally, each shaft is fitted with a pulley which the belt is situated in. As the drive pulley turns, the belt transmits power to the driven pulley. The figure on the following page shows two views of a dual V-belt drive arrangement.

V-Ribbed Belts

V-ribbed, or Poly-V belts, are flat belts with a series of ribs running longitudinally along the driving face that fit into grooves in a sheave. These belts are relatively thin with a well-supported tensile member. They are thinner and weigh less than V-belts. Poly-V belts are a single unit utilizing an uninterrupted, full width tensile member which is completely supported. The drive load is evenly distributed across the width of the sheave, equalizing belt stress. A Poly-V belt also resists seating in the grooves, so speed ratios remain more consistent and output speed remains more uniform. These belts utilize the entire width of the pulley face, allowing more compact drives. Poly-V belts allow narrowing mounting clearances, need less center distance adjustment and require less take-up for tensioning. Narrow sheaves of smaller diameter can be used without sacrificing power capacity while reducing weight and increasing efficiency.

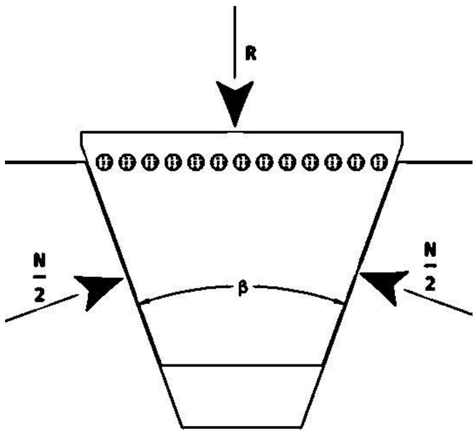
<u>Advantages</u>

- Combines high power capacity of V-belts with the flexibility of flat belts
- With a thin cross section and low weight, high speed ratios (up to 40:1)
- Excel on small sheaves, at high speeds and with reverse bends
- Generally run smoother than V-belts
- Each belt is a single unit, no differential driving occurs (when load is carried unequally in a multi-belt drive)
- No separate belts to turn over, slip or interfere with each other
- Never any matching problems, the a V-ribbed belt is manufactured as a single unit
- Track properly without the need for guides, flanges, crowns or deep grooves

V-Belts

The V-belt drive friction depends on several factors:

- Total tension in the drive, including static and centrifugal tension
- Coefficient of friction between the belt and pulley
- Angle of contact, dependent on pulley diameters and the center distance
- Centrifugal force lifting the belt off the pulley, produced by the rotation of the pulley
- Angle of the "V" in the pulley, which wedges the belt in place



Wedging Effect

- V-Belts largest advantage over flat-belt drives is the utilization of wedging to increase the total friction between the belt and pulley without increasing the hub load or effective tension.
- Tensioning a V-belt will cause the tensile members of the belt to exert resulting force **R** on the body of the belt. The body of the belt in turn exerts a total normal force **N** on the pulley groove. But because the bottom of the belt does not contact the bottom of the pulley, the sidewalls are where the force is exerted.
- Balancing the forces gives: $R = N \sin\frac{\beta}{2}$
    - Therefore, $R' = \frac{N}{\sin^{\beta}/_{2}}$ For a 38° included sheave angle, N ≈ 2.92xR

- For a flat belt drive, the friction between the belt and pulley for a small segment of belt is $F = \mu \times R$, while for a V-belt/sheave with a 38° angle, the friction would be $F = 2.92 \times \mu \times R$
    - The same hub load and stress in the tensile members gives V-belts nearly 3x the friction of flat belts.
        - For a 38° angle, assuming all other characteristics are the same.

Advantages of V-Belts

- Reduced shock and vibration transmission
- Quite and require no lubrication
- Much cheaper, lighter, and cleaner than chains
- Somewhat tolerant to misalignment and abuse
- Easy installation and minimal maintenance
- Simple to change shaft speed by changing pulley diameters $\frac{Speed\ of\ Driver}{Speed\ of\ Driven} = \frac{Diameter\ of\ Driven}{Diameter\ of\ Driver}$
- Flexible equipment design, many sizes and cross sections available
- High efficiency, up to 98%
- Extremely wide horsepower and speed ranges
- Prevent severe power overload; can be used as a method of clutching
- Belts and pulleys wear gradually, allowing for simple preventative maintenance

Timing Belts

Positive Engagement

- Timing belts, or synchronous belts, were developed as a compromise between chain and flat belt drives. Modern timing belts offer high power ratings coupled with minimal maintenance.
- Because they do not depend on friction to provide grip between the belt and the sprockets, timing belts require less tension than V-belts. This leads to lower bearing loads and reduced stress within the belt.

Advantages

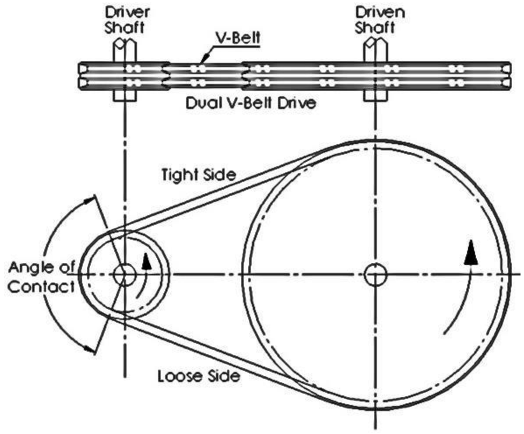
- Do not slip under shock or pulsating loads
- Dust, oil and grease do not lead to slip
- Useful for applications where a fixed speed ratio or timing is required
- Virtually no elongation due to wear
- Significantly quieter than chains
- Thinner than V-belts, meaning less energy gets used bending the belt around pulleys
- Several tooth profiles have been developed for a range of applications
- Can be made with teeth on both sides for drives where the rotation direction is to be reversed without belt twisting
- Allow for precise rotary positioning of shafts relative to one another
- Unlike chains, timing belts require no lubrication (reduced recurring costs) and minimal maintenance
- No need for tensioning devices, if properly installed, initial tension will suffice for the life of the belt

Lateral Travel

Side to side movement of a timing belt, or “tracking” can be caused by several factors and is the reason flanges are used.

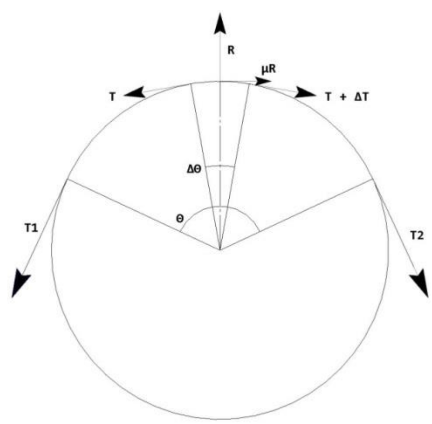
- Misalignment - a belt will usually “climb” to the position on the pulleys where it is tightest (the points farthest apart)
- Tension - lateral travel can be altered by adjusting tension
- Pulley Diameter - belts tend to track with more force on smaller pulleys
- Plane Location - vertical drives are more likely to track
- Belt Length/Width Ratio - the higher this ratio, the less likely the belt will move side to side
- Direction of Cord Lay - the direction the cords were wrapped around the mold will affect belt movement
- Direction of Cord Twist - the cords in the belt are twisted in either a “S” or “Z” construction

Transmission of Force - V-Belts and V-Ribbed Belts



If a belt is tensioned between two pulleys which are free to turn, the tension along the length of the belt is constant and equal to “T”. However, when transmitting torque, the driven pulley will resist the motion and the driver pulley will have to pull on the “Tight Side” in order to exert torque on the driven pulley. The tension in the “Tight Side” can be called “T_1”. The total tension “T_T” must remain constant. Thus, if a “Tight Side” is formed a “Loose Side” must also be formed. This “Loose Side” tension can be called “T_2”. Therefore, if “T_T” remains constant then $T_T = T_1 + T_2$ at all times. The effectiveness of the drive depends on maintaining frictional contact between the belt and the pulleys.

- Power is the rate of doing work. Power transmission by a V-belt is dependent on the effective tension “T_e” and the belt speed. The general power equation is $P = v(T_1 - T_2) = vT_e$
- When the limiting friction is developed around the arc of contact, the belt can transmit the maximum torque to the pulley or vice versa.
- “T_1” & “T_2” are the tight and loose side tensions, respectively. “R” is the resulting force perpendicular to the belt at a point. μ is the coefficient of friction. “T” is the tension at a given point. For simplification, “μR” is not accounting for the wedging effect of a belt.
- Analyzing the forces in the diagram tangentially for a small ΔΘ under low speed no-slip conditions leads to the following:
  - $\mu R = (T + \Delta\mathrm{T}) \cos\frac{1}{2}\Delta\Theta - \mathrm{T}\cos\frac{1}{2}\Delta\Theta$
  - Thus, taking the limit as ΔΘ→0 gives us $\mu R = \Delta\mathrm{T}$

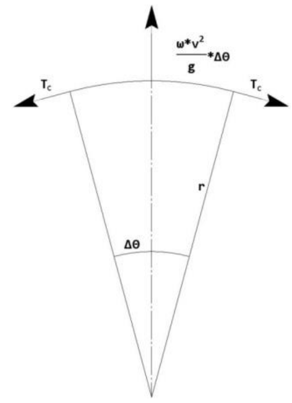


- Radially:
  - $R = (T + \Delta\mathrm{T}) \sin\frac{1}{2}\Delta\Theta + T\sin\frac{1}{2}\Delta\Theta$
  - As ΔΘ→0, $\sin\Delta\Theta = \Theta$ and ΔT = 0
  - Taking the limit, $R = T\Delta\Theta$
  - Substituting with $\mu R = \Delta\mathrm{T}$, $\frac{\Delta T}{T} = \mu\Theta$
  - Integrating over the entire Angle of Contact (θ), $\ln\left(\frac{T_1}{T_2}\right) = \mu\Theta$ which gives $\frac{T_1}{T_2} = e^{\mu\Theta}$ This formula governs the amount of power which can be transmitted before the belt slips.
- This is the effective tension ratio between the tight and loose sides of the belt. For a belt drive between pulleys of different diameters, the angle of contact of the smaller pulley should be used.

Centrifugal Force & Tension

- In many cases, the speeds at which V-belts are operated can cause significant centrifugal force.
- Centrifugal force represents the effects of inertia which arise in connection with rotation and are experienced as an outward force away from the center of rotation.
- Where ω is the weight of a unit length for a belt of a certain cross section and is equal to ρ*A ;   v is the belt speed.
- The force on a belt element of length (ΔΘ*r) is given by $F_C = \rho * A * v^2$

  $F_C = \left[\frac{w*r*\Delta\Theta}{g}\right] \times \frac{v^2}{r} = \frac{w*v^2*\Delta\Theta}{g}$

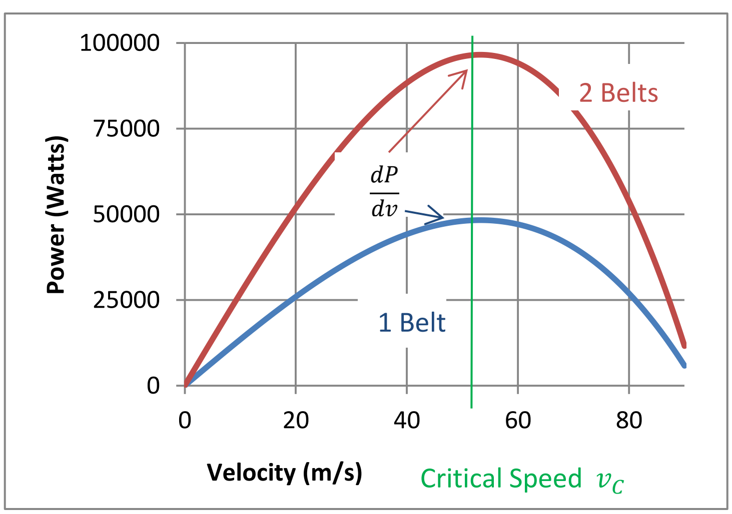


- The forces must balance, so

$$2 \times T_C \times \sin\tfrac{1}{2}\Delta\Theta = F_C = \rho * A * v^2 \quad \text{As } \Delta\Theta \to 0,\ \sin\tfrac{1}{2}\Delta\Theta = \frac{\Theta}{2}$$

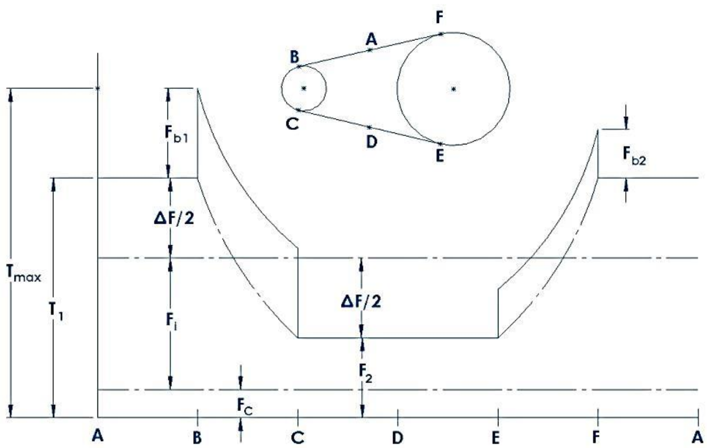
$$\text{Therefore, } T_C = \frac{w*v^2}{g} = \frac{\rho \times A \times v^2}{\Theta}$$

- These additional forces must be accounted for when designing a drive.
  - The centrifugal force F_C will reduce the amount of friction between the belt and the pulley, which could lead to excessive slip in extreme situations.
  - The centrifugal tension T_C will increase the total tension in the belt. If a belt was installed with too much tension, this additional tension can lead to premature belt failure due to excessive stress in the tensile members.
- Maximum power can be limited by centrifugal force.
  - As previously mentioned, the power transmitted by a belt is $P = v(T_1 - T_2) = vT_e$ Additionally, $\frac{T_1}{T_2} = e^{\mu\Theta}$ Solving $\frac{T_1}{T_2} = e^{\mu\Theta}$ for T_2, $T_2 = \frac{T_1}{e^{\mu\Theta}} = T_1 e^{-\mu\Theta}$ Substituting T_2 into the power equation gives us $P = v(T_1 - T_1 e^{-\mu\Theta}) = vT_1(1 - e^{-\mu\Theta})$
  - The approximate power equation when considering centrifugal force is $P = v(T_1 - \rho A v^2)(1 - e^{-\mu\Theta})$ Using this equation and plotting the power vs speed for a nominal set of parameters results in the graph below.
- At zero speed, the power is obviously zero. Initially, one might think that, assuming a constant torque, as the speed increases, the power able to be delivered will continue to increase. However, the centrifugal force of the belt will begin to decrease the effective normal force between the belt and the pulley, resulting in less traction. The belt will start to slip.
- Taking the derivative at the critical speed, $\frac{dP}{dv} = 0 = (T_1 - \rho A {v_C}^2)(1 - e^{-\mu\Theta})$
- Therefore, $v_C = \sqrt{T_1/3\rho A}$ the speed at which maximum power can be delivered.
- Critical speed must be calculated before using $P_{max} = \frac{2}{3}(v_C T_1)(1 - e^{-\mu\Theta})n$ where n is the number of belts.
- Note that these factors are usually only a concern at very high speeds.
- While high speeds do not generally reduce the ability of a timing belt to transmit power, high tensions can result from large centrifugal forces.



Creep and Shear

- Creep when referring to belt drives is not the same as when used in regards to other engineering applications. The change in length of the belt over time is called stretch. Creep is not the same as slip. Creep is defined as a loss of driven speed as the result of alternate lengthening and shortening of each belt segment as it cycles through the tight and loose side tensions.
- Whenever a belt passes around a pulley with different entering and exiting tensions, i.e. any driving or driven pulley, belt creep occurs. The belt will be slightly longer on the tight side than on the loose side. As the belt travels throughout the arc of contact, the length of a belt segment will slowly change from the tight side length to the loose side length as tension decreases. This motion relative to the pulley is creep and leads to some sidewall wear.
- A critical assumption of creep theory is that the belt is elastic enough to allow for large enough differences between tight and loose side nominal segment lengths that any shear stress within the belt can be considered negligible. (Developed for round belts with significant stretch and flat belts thin enough for minimal shear stress.)
- Shear theory assumes that the stretch experienced by a belt as it alternates between tight and loose side tensions is negligible compared to the shear stress within the belt. (Developed as a result of high strength tensile members limiting cordline stretch.)



- The diagram to the left shows the stress within the tensile members as a belt travels around the pulleys shown. "F_C" is the tension induced by centrifugal force. "F_i" is the initial static tension of the belt. "ΔF" is tension difference induced by the transmission of torque. Note that the maximum stress in the tensile members occurs at the entry point (tight side) of the small pulley.
- The solid curve shows the stress as described by shear theory while the dashed curve shows the stress described by creep theory. Point "A" is the tight side; point "D" is the loose side. All other points are the tangential entry or exit points of the belt on the pulley. The vertical jumps in stress on the shear stress curve indicate the additional stress imposed on the members of the belt when conforming to the curve of the pulley assuming relatively inextensible tensile members. Those tension forces are balanced within the belt by compression stress within the body of the belt. Note that the stress imposed by the smaller pulley is larger due to the smaller radius the belt must conform to. Both cases result in fluctuating stresses within the belt, leading to fatigue and eventual failure.

Fatigue

- As indicated above, the maximum stress in the belt is at the tight side entry point of the small pulley. The stress is a combination of the tight side tension and the bending stresses.
  - Elastic bending theory tells us that $\sigma_{bending\ max} = {y_{max} * E}/{R}$ where y_{max} is the maximum distance to the neutral axis, E is the elastic modulus, and R is the radius of the curve.
- The cyclical stress (seen in the above diagram) is what leads to the fatigue failure seen in belts. During each revolution of the belt, each infinitesimal belt segment goes through a double-peaked fatigue cycle (Peaks at B & F, minimum from C-E). Generally, cyclically loaded components follow an approximately log-log linear relationship between the life and load of the component.
- Doubling the load a belt carries will not lead to half of the original life. The new life of the belt could be as little as 5%-10% of the original.